

Factory by Viking

After reading the preamble, Gregson thought this puzzle would be extremely difficult. After solving the first half dozen clues very quickly, Gregson revised that opinion and thought the puzzle would be almost trivially easy. Then, after grinding to a halt, Gregson revised that opinion again. Unusually for a mathematical, this puzzle did not seem to get easier as the grid was filled in. Progress was fairly steady, requiring a number of different techniques to keep the solution moving, which is a tribute to the setter's skill.

Before starting to produce these reviews, Gregson confesses to having used any and every artificial aid available from calculators to spreadsheets to bespoke software. Gregson then decided to attempt them in the manner avowed possible by the editors – pencil, paper and calculator. The only concession has been the use of pre-prepared lists and tables of numbers and their properties, such as those to be found in Derek Arthur's excellent "Solving Mathematical Crosswords", available on the Listener website, which Gregson cannot recommend highly enough. Indeed, something tells Gregson that Viking is very likely to have used these lists in setting the puzzle!

All in all a highly fitting tribute to Rhombus – and sobering to realize that solvers of Rhombus' original puzzle would almost certainly have done it without even a calculator.

Gregson's route to the solution was as follows. As usual with the mathematical puzzles, there will be other, no doubt quicker, routes.

aA 17 is prime \rightarrow answer is P^{16}
only possibility is $P = 3 \rightarrow aA = 43046721$

g 7 is prime \rightarrow answer is P^6
only possibility is $P = 3 \rightarrow g = 729$

b cross checking answers give $b = 32 = 2^5$ with 6 factors

aD 7 is prime \rightarrow answer is P^6
only possibility is $P = 19 \rightarrow aD = 47045881$

e 3 is prime \rightarrow answer is P^2
only possibility is $P = 17 \rightarrow e = 289$

x 5 is prime \rightarrow answer is P^4
only possibility is $P = 3 \rightarrow x = 81$

s 5 is prime \rightarrow answer is P^4
only possibility is $P = 5 \rightarrow s = 625$

- m 3 is prime \rightarrow answer is P^2
possibilities are $P = 13$ (169) or $P = 17$ (289) or $P = 23$ (529)
but d has 2 factors $\rightarrow d$ is prime and cannot end with 2 or 5
hence $P = 13 \rightarrow m = 169$
- f 5 is prime \rightarrow answer is P^4
possibilities are $P = 59$ (12117361) or $P = 61$ (13845841)
this means $k = 82$ or 83
but k has 4 factors $\rightarrow k = 82 \rightarrow f = 12117361$
- h 27 factors \rightarrow answer is P^{26} or P^2Q^8 or $P^2Q^2R^2$
range and crosschecking rule out P^{26} and P^2Q^8 leaving $P^2Q^2R^2$
only possibility is $P = 2$ $Q = 3$ $R = 5 \rightarrow h = 900$
- i 3 is prime \rightarrow answer is P^2
any 3-digit square of a prime must end with 1 or 9
this means j must be 16? or 96?
- j 10 factors \rightarrow answer is P^9 or PQ^4
 P^9 is impossible so answer is PQ^4
only possibility is $P = 2$ $Q = 3 \rightarrow j = 162$
- c 3 is prime \rightarrow answer is P^2
only possibilities are $P = 211$ (44521) or $P = 223$ (49729)
this means i starts with 4 or 9
- i 3 is prime \rightarrow answer is P^2
only possibility matching 4?1 or 9?1 is $P = 31 \rightarrow i = 961$
this means $c = 49729$
- d crosschecking answers give $d = 661$ which has two factors
- o 15 factors \rightarrow answer is P^{14} or P^2Q^4
 P^{14} is impossible so answer is P^2Q^4
 o is 2?2?1 so all factors are odd and $\neq 5$
range constraint means that Q must be 3 or 7
 $Q = 7$ is impossible $\rightarrow Q = 3$
only possibility is $P = 19$ $Q = 3 \rightarrow o = 29241$
- w 14 factors \rightarrow answer is P^{13} or PQ^6
 P^{13} is impossible \rightarrow answer is PQ^6
range constraint means that $Q = 2$, hence both w and q are even
- q 9 factors \rightarrow answer is P^8 or P^2Q^2
 P^8 is impossible \rightarrow answer is P^2Q^2
 q is even $\rightarrow P = 2$
only possibility is $P = 2$ $Q = 11 \rightarrow q = 484$
- tA 6 factors \rightarrow answer is P^5 or PQ^2

- P^5 is impossible \rightarrow answer is PQ^2
only possibility is $P = 43$ $Q = 3$ $tA = 387$
- tD 10 factors \rightarrow answer is P^9 or PQ^4
 P^9 is impossible \rightarrow answer is PQ^4
only possibilities are $P = 19$ $Q = 2$ $tD = 304$ and $P = 23$ $Q = 2$ $tD = 368$
this means middle digit of w is 0 or 6
- p 3 is prime \rightarrow answer is P^2
only possibilities are $P = 307$ $p = 94249$ or $P = 311$ $p = 96721$ or $P = 313$ $p = 97969$
this means first digit of w is 2 or 7 or 9
- w 14 factors \rightarrow answer is P^{13} or PQ^6
 P^{13} is impossible \rightarrow answer is PQ^6
crosschecked digits give possibilities as 204 264 704 764 904 964
only 704 has 14 factors \rightarrow $P = 11$ $Q = 2$ $w = 704$
- tD crosschecked digits give $tD = 304$
- p crosschecked digits give $p = 96721$
- z 10 factors \rightarrow answer is P^9 or PQ^4
 P^9 is impossible \rightarrow answer is PQ^4
only possibility is $P = 31$ $Q = 2$ $z = 496$
- r 15 factors \rightarrow answer is P^{14} or P^2Q^4
 P^{14} is impossible \rightarrow answer is P^2Q^4
r must be divisible by 8 \rightarrow $Q = 2$
only possibility is $P = 59$ $Q = 2$ $r = 55696$
- lA 6 factors \rightarrow answer is P^5 or PQ^2
 P^5 is impossible \rightarrow answer is PQ^2
possibilities are $P = 23$ $Q = 3$ $lA = 207$ or $P = 3$ $Q = 13$ $lA = 507$
this means first digit of lD is 2 or 5
- n 24 factors, $n = 4?0$
possibilities with 24 factors are 420 and 480
this means second digit of lD is 2 or 8
- lD 9 factors \rightarrow answer is P^8 or P^2Q^2
 P^8 is impossible \rightarrow answer is P^2Q^2
lD is divisible by 5 \rightarrow $P = 5$
crosschecked digits give possibilities 225 285 525 585
only possibility is $P = 5$ $Q = 3$ $lD = 225$
- lA crosschecked digits give $lA = 207$
- n crosschecked digits give $n = 420$
- uA 3 is prime \rightarrow answer is P^2

possibilities are $P = 11$ $uA = 121$ and $P = 23$ $uA = 529$
this means first digit of uD is 1 or 5

uD 16 factors \rightarrow answer is P^{15} or PQ^7 or PQR^3 or P^3Q^3 or $PQRS$
 P^{15} and PQ^7 are impossible \rightarrow answer is PQR^3 or P^3Q^3 or $PQRS$
only possibility is $P = 2$ $Q = 3$ $R = 5$ $S = 17$ $uD = 510$

uA crosschecked digits give $uA = 529$
this means first digit of v is 9

y 10 factors \rightarrow answer is P^9 or PQ^4
possibilities are $P = 2$ $y = 512$ or $P = 37$ $Q = 2$ $y = 592$
this means middle digit of v is 1 or 9

v 2 factors \rightarrow v is prime
crosschecked digits give $v = 91?$ or $99?$
possibilities are $v = 911$ or 919 or 991 or 997
this means fourth digit of B is 1 or 7 or 9 (although 1 would be ambiguous)

A 6 factors \rightarrow answer is P^5 or PQ^2
 P^5 is impossible \rightarrow answer is PQ^2
possibilities are $P = 23$ $Q = 2$ $A = 92$ or $P = 2$ $Q = 7$ $A = 98$ or $P = 11$ $Q = 3$ $A = 99$
this means seventh digit of B is 2 or 8 or 9

B 9 factors \rightarrow answer is P^8 or P^2Q^2
 P^8 is impossible \rightarrow answer is P^2Q^2
 B ends with 1 \rightarrow neither P nor Q is 2 or 5
the squares of all primes other than 2 and 5 end with 1 or 9
 B ends with 1 \rightarrow either P^2 and Q^2 both end with 1 or both end with 9
thus either both P and Q end with 1 or 9 or both P and Q end with 3 or 7
 B starts with 10 so range constraints mean P (say) ≤ 53 with corresponding

limits on Q

all this together gives possibilities

$P = 3$ $Q = 1063$ or 1087 or 1093 or 1097 or 1103

$P = 7$ $Q = 457$ or 463 or 467

$P = 17$ $Q = 193$

$P = 41$ $Q = 79$

only possibility which matches crosschecked digits is $P = 41$ $Q = 79$ $B = 10491121$

A crosschecked digits give $A = 92$

v crosschecked digits give $v = 919$

y crosschecked digits give $y = 512$