One More Or Less

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Solution

- A good starting point are the Fibonacci numbers, those being 144, 233, 377, 610 and 987. Looking at A, a and C along with e means that we need only consider the first three on the list as the last two would give an negative answer for e.

- The calculation of the digit product is done with the entries which are one more or one less than their type. So for 144 we take 143 and 145, for 233 we use 232 and 234 and for 377 we take 376 and 378. The associated squares are 9 [entry 10] and 16 [entry 15 or 17] for 144, 25 [entry 24 or 26] for 233 and 36 [entry 35 or 37 for 377.

- This yields for A + a

Sq/Fib	143	145	232	234	376	378
10	153	155				
15	158	160				
17	160	162				
24			256	258		
26			258	260		
35					411	413
37					413	415

And for the digit product the following

	143	145	232	234	376	378
10	15	25				
15	40	0				
17	0	12				
24			60	80		
26			80	0		
35					4	12
37					12	20

We need the tens digit of the Fibonacci number a to be the same as that for the digit product or one more or one less. Only 15, 143 and 40 fit the bill. So A = 15, a =

143 and C = 41.

- e is 41 – 15 = 26 so the entry is 25 or 27.

- E must be the prime 31 which is entered as 32.

- Now tackle the D, d, F triple. Since D is prime it will be entered with an even units digit so we need only consider 233 and 610 for the Fibonacci number. Our calculations will involve 232 and 234 along with 609 and 611. The primes for the former set can end in 1 or 3 and for the latter set only in 7. This yields the following for the D + d.

	12	22	32	42	52	62	72	82
232	244	254	264	274	284	294	304	314
234	246	256	266	276	286	296	306	316

	16	36	46	66	96
609	625	645	655	675	705
611	627	647	657	677	707

And for the digit product the following

	12	22	32	42	52	62	72	82
232	32	40	48	56	64	72	0	12
234	48	60	72	84	96	108	0	18

	16	36	46	66	96
609	60	120	150	210	0
611	84	168	210	294	0

This time we need the units digit of the product to be one more or one less than the tens digit of the Fibonacci number. We can eliminate the 610 possibility as that is entered as 609 or 611. We have a number of possibilities (232, 12, 32 [entry 33]), (232, 52, 64 [entry 63]), (232, 62, 72 [entry 73]), (232, 82, 12 [entry 13]), (234, 32, 72 [entry 73]) and (234, 42, 84 [entry 83]). We can eliminate the penultimate set as 32 is already in the grid and the last set as F – D is 41 and can't be made to end in a four as required. Of the remaining three sets only the first set will work as the other two can be eliminated from c. So F is 32 with the entry as 33, d is 233 with the entry as 232 and D is 11 or 13 it doesn't matter which as the entry is 12. By calculation H is 21 with the entry as 22.

- The calculation for c gives 80 or 82 so the entry must be 81.

- B/b must start with a 1. So the triangular number is 190 with the entry as 189.

- G is a multiple of 96 and more than 810. There are two possibilities 864 and 960. The former can be discounted as its middle digit has to be 5 or 7. The latter 960 becomes 959 on entry. So e is 25 and c is 80 entered as 81.

- Finally b is 1148 by calculation and entered as 1149.

The final grid is as follows.

1	5	1	8	9
4	1	1	1	2
3	2	4	3	3
9	5	9	2	2

The answers/entries are

A 16/15   B 190/189   C 40/41   D 11,13/12   E 31/32   F 32/33

G 960/959   H 21/22   a 144/143   b 1148/1149 c 80/81   d 233/232

e 26/25.